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K-meson neutrinoless double muon decay as a probe of neutrino masses and mixings

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Abstract

Recently an upper bound on the rate of the lepton number violating decay $K^+ \to \mu^+\mu^+\pi^-$ has been significantly reduced by the E865 experiment at BNL and further improvement is expected in the near future.

We study this process as a possible source of information on neutrino masses and mixings. We find that it is insensitive to the light(eV domain) and heavy(GeV domain) neutrinos. However due to the effect of a resonant enhancement this decay is very sensitive to neutrinos ν_j in the mass region $245\text{MeV} \leq m_{\nu_j} \leq 389\text{MeV}$. At present experimental sensitivity we deduce a new stringent limit on the mixing matrix element $|U_{\mu j}|^2 \leq (5.6 \pm 1)10^{-9}$ for neutrino masses in this region.

1 Introduction

Nowadays there are sufficient indications to believe that neutrinos are massive particles mixing with each other. These indications come from both experimental and theoretical sides. The solar neutrinos deficit, the atmospheric neutrino anomaly and the results of the LSND neutrino oscillation experiment, all can be explained in terms of neutrino oscillations implying non-zero neutrino masses and mixings. A breakthrough in this direction has been achieved by the latest SuperKamiokande measurements of the up-down asymmetry of multi-GeV atmospheric muon events [1]. These results give a convincing evidence that muon neutrinos oscillate to the other neutrino species. On the theoretical side there are also many indications in favor of non-zero neutrino masses following from almost all phenomenologically viable models of the physics beyond the standard model(SM). These models typically predict Majorana type neutrino masses suggesting that neutrinos are truly neutral particles.

The neutrino oscillation searches fix the neutrino mass square difference $\delta m_{ij}^2 = m_i^2 - m_j^2$ and the neutrino mixing angles, leaving the overall mass scale and the CP-phases arbitrary. Since the latter has no effect on neutrino oscillations, the important question of whether neutrinos are Majorana or Dirac particles cannot be answered by these searches. Thus we need additional means to study these properties of neutrinos.

Majorana masses violate the total lepton number conservation by two units $\Delta L = 2$. Therefore lepton number violating($\not{L}$) processes represent a most appropriate tool to address the question of Majorana nature of neutrinos. A celebrated example of $\not{L}$ -process, most developed experimentally and theoretically, is the neutrinoless nuclear double beta $(0\nu\beta\beta)$ decay (for review see [2]). The $0\nu\beta\beta$-experiments achieved unprecedented sensitivity to the so called effective Majorana neutrino mass $\langle m_\nu \rangle_{ee} = \sum U_{ei}^2 m_{\nu i} \leq 0.2\text{eV}$ [3], where $m_{\nu i}$ and U_{ei} are the neutrino masses and mixing matrix elements. In the presence of only light neutrinos the effective Majorana mass coincides with the entry of the neutrino mass matrix $\langle m_\nu \rangle_{ee} = M_{ee}^{(\nu)}$.

Needless to say that any other information on the Majorana neutrino mass matrix would be important. This information can be inferred from studying other $\not{L}$ processes. Besides $0\nu\beta\beta$-decay, other $\not{L}$ processes have been studied in the literature in this respect, from both theoretical and experimental sides. Among them there are the decay $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ [4, 5, 6, 7, 8], nuclear muon to positron conversion, trimuon production in neutrino-nucleon scattering [9], the process $e^+p \to \bar{\nu} l_1^+ l_2^+ X$, relevant for HERA [10], as well as direct production of heavy Majorana neutrinos at various colliders [11]. Unfortunately sensitivities of the current experiments searching for these processes are much less than in the previous case of $0\nu\beta\beta$-decay. The analysis made in the literature [6, 12] leads to the conclusion that these processes, except $0\nu\beta\beta$-decay, can hardly be observed experimentally. This analysis relies on the current neutrino oscillation data, and on certain assumptions about the neutrino spectrum. In the present paper we concentrate on the neutrinoless double muon decay of kaon $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$. We will show that despite the above conclusion being true for contributions of the neutrino states much lighter or much heavier than the typical energy of the $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay, there is still a special window in the neutrino sector which can be efficiently probed by searching for this process. This window is in the neutrino mass range $245\text{MeV} \leq m_{\nu_j} \leq 389\text{MeV}$, where the s-channel neutrino contribution to the $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay is resonantly enhanced, therefore making this decay very sensitive to the neutrinos in this mass domain. If neutrinos with masses in this region exist in nature we can extract stringent limits on their mixing with ν_μ. We derive these limits from the constraints on the branching ratio of $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ recently obtained by E865 experiment at BNL [13].

2 $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay in Standard Model with Majorana neutrinos

In the SM extension with Majorana neutrinos there are two lowest order diagrams, shown in Fig.1 which contribute to $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay. These diagrams were first considered long ago in Refs.[4, 5]. Here we are studying previously overlooked aspects of this decay. We concentrate on the s-channel neutrino exchange diagram in Fig. 1(a) which plays a central role in our analysis. The t-channel diagram in Fig.1(b) requires in general a detailed hadronic structure calculation. In Ref. [5] this diagram was evaluated in the Bethe-Salpeter approach and shown to be an order of magnitude smaller than the diagram in Fig.1(a), for light and intermediate mass neutrinos. As we will see in the neutrino mass domain of our main interest the diagram in Fig.1(a) absolutely dominates over the t-channel diagram in Fig.1(b), independently of hadronic structure.

A contribution of the factorizable s-channel diagram in Fig.1(a) can be calculated in a straightforward way without referring to any hadronic structure model. A final result for the $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay rate is given by

$$\Gamma(\mathrm{K}^+ \to \mu^+\mu^+\pi^-) = c\int\limits_{s_1^-}^{s_1^+} ds_1 \sum_k \left|\frac{U_{\mu k}^2 m_{\nu k}}{s - m_{\nu k}^2}\right|^2 G(\frac{s_1}{m_K^2}) + \tag{1}$$

$$2\frac{c}{m_K^2}\mathrm{Re}[\int\limits_{s_1^-}^{s_1^+} ds_1 \frac{U_{\mu k}^2 m_{\nu k}}{s_1 - m_{\nu k}^2}\int\limits_{s_2^-}^{s_2^+} ds_2 \left(\frac{U_{\mu k}^2 m_{\nu k}}{s_2 - m_{\nu k}^2}\right)^* H(\frac{s_1}{m_K^2}, \frac{s_2}{m_K^2})].$$

The unitary mixing matrix U_{ij} relates $\nu'_i = U_{ij}\nu_j$ weak ν' and mass ν neutrino eigenstates. The numerical constant in Eq. (1) is $c = (G_F^4/32)(\pi)^{-3} f_\pi^2 f_K^2 m_K^5 |V_{ud}|^2 |V_{us}|^2$, where $f_K = 1.28 f_\pi$, $f_\pi = 0.668 m_\pi$ and $m_K = 494$MeV is the K-meson mass. The functions $G(z)$ and $H(z_1, z_2)$ in Eq. (1) after the phase space integration can be written in an explicit algebraic form

$$G(z) = \frac{\phi(z)}{z^{-2}}\left[h_{+-}(z) - x_\mu^2)h_{--}(z) - x_\pi^2 h_{-+}(z))\right]\left[x_\mu^2 + z - (x_\mu^2 - z)^2\right] \quad (2)$$
$$H(z_1, z_2) = h_{--}(z_1)h_{--}(z_2) + z_\pi^2[r_+ - x_\mu^2 t(z_1, z_2, 1)] - r_-(z_1 z_2)t(z_1, z_2, x_\mu).$$

Here we defined $x_i = m_i/m_{_K}$ and $h_{\pm\pm}(z) = z \pm x_\pi^2 \pm x_\mu^2$, $r_\pm(z_1 z_2) = z_1 z_2 - x_\pi^2 \pm x_\mu^4$, $t(z_1, z_2, z_3) = z_1 + z_2 - 2z_3^2$, $\phi(y) = \lambda^{1/2}(1, x_\mu^2, z)\lambda^{1/2}(z, x_\mu^2, x_\pi^2)$ with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$. The integration limits in Eq. (1) are

$$s_1^- = m_{_K}^2(x_\pi + x_\mu)^2, \quad s_1^+ = m_{_K}^2(1 - x_\mu)^2, \quad (3)$$
$$s_2^\pm = 2m_{_K}y^{-1}\left[2y(1 + x_\mu^2) - (1 + y - x_\mu^2)h_{-+}(y) \pm \phi(y)\right]$$

with $y = s_1/m_{_K}^2$,

First we assume that neutrinos can be separated into light ν_k and heavy N_k states, with masses $m_{\nu i} << \sqrt{s_1^-}$ and $\sqrt{s_1^+} << M_N$. Then the Eq. (1) can be approximately written as

$$\Gamma(\mathrm{K}^+ \to \mu^+\mu^+\pi^-) = |\langle m_\nu\rangle_{\mu\mu}|^2\, m_{_K}^{-1}\mathcal{A}_\nu + \left|\langle M_N^{-1}\rangle_{\mu\mu}\right|^2 m_{_K}^3\mathcal{A}_N \quad (4)$$
$$+\mathrm{Re}\left[\langle m_\nu\rangle_{\mu\mu}\langle M_N^{-1}\rangle_{\mu\mu}\right] m_{_K}\mathcal{A}_{\nu N},$$

where the average neutrino masses are determined in the standard way

$$\langle m_\nu\rangle_{\mu\mu} = \sum_{k=light} U_{\mu k}^2 m_{\nu k}, \quad \langle M_N^{-1}\rangle_{\mu\mu} = \sum_{k=heavy} U_{\mu k}^2 M_{Nk}^{-1}. \quad (5)$$

The following comments are in order. If the approximate formula (4) is used for extracting limits on the average neutrino masses from the experimental data with the result: $\langle m_\nu\rangle \leq Exp(\nu)$, $\langle M_N^{-1}\rangle \leq Exp(N)$ then the following consistency conditions must be satisfied $Exp(\nu), Exp(N) << \sqrt{s_1^\pm} \sim m_{_K}$. Otherwise the so derived limits are not applicable to $\langle m_\nu\rangle$, $\langle M_N^{-1}\rangle$ as it is in Refs. [7, 8, 10]. Similar conditions take place for the other $\not{L}$ processes. In this case $m_{_K}$ is to be changed to the corresponding characteristic energy.

The dimensionless coefficients in (4) are $\mathcal{A}_\nu = 9.2{\cdot}10^{-31}$, $\mathcal{A}_N = 1.6{\cdot}10^{-31}$, $\mathcal{A}_{\nu N} = 3.9 \cdot 10^{-31}$. With these numbers we can estimate the current upper bound on the $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay rate from the experimental data on other processes.

Atmospheric and solar neutrino oscillation data, combined with the tritium beta decay endpoint, allow one to set upper bounds on the masses of the known three neutrinos [14] $m_{e,\mu,\tau} \leq$ 3eV. Thus in the three neutrino scenario one gets $\langle m_\nu \rangle_{\mu\mu} \leq$ 9eV. In this case we derive from Eq. (4) the following branching ratio

$$\mathcal{R}_{\mu\mu} = \frac{\Gamma(\mathrm{K}^+ \to \mu^+\mu^+\pi^-)}{\Gamma(\mathrm{K}^+ \to all)} \leq 3.0 \cdot 10^{-30} \quad 3 \; light \; neutrino \; scenario. \tag{6}$$

Assuming the existence of heavy GeV mass neutrinos N, we may obtain an upper bound on $\Gamma(\mathrm{K}^+ \to \mu^+\mu^+\pi^-)$ using the current LEP limit on heavy stable neutral leptons $M_N \geq 39.5$ MeV [15] which leads to $\langle M_N^{-1} \rangle_{\mu\mu} \leq n\,(39.5\mathrm{GeV})^{-1}$, where n is the number of heavy neutrinos. This limit being substituted in Eq. (4) results in the upper bound

$$\mathcal{R}_{\mu\mu} \leq 2.0 \cdot 10^{-19} \quad 3 \; light + 1 \; heavy \; neutrino \; scenario \tag{7}$$

Direct searches for $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay by E865 experiment at BNL [13] give

$$\mathcal{R}_{\mu\mu} \leq 3.0 \cdot 10^{-9} \quad (90\%\mathrm{CL}, \quad \mathrm{Ref.}[13]) \tag{8}$$

Comparison of this experimental bound with the theoretical predictions in Eqs. (6), (7) clearly shows that both cases are far from being ever detected. On the other hand experimental observation of $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay at larger rates would indicate some new physics beyond the SM, or, as we will see, the presence of an extra neutrino state ν_j with the mass in the hundred MeV domain. Let us note that such neutrinos are not excluded by the LEP neutrino counting experiments measuring the Z-boson invisible width. These experiments set limits not on the number of light massive neutrinos but on the number of active neutrino flavors $N_\nu = 3$. Thus extra massive neutrino states ν_j can appear as a result of mixing of the three active neutrinos with certain number of sterile neutrinos. These massive neutrinos are searched for in many experiments [16]. The ν_j states would manifest themselves as a peaks in differential rates of various processes, and can give rise to significant enhancement of the total rate if their masses lie in an appropriate region.

3 Constraints on neutrinos in the hundred MeV mass domain

Assume there exists a massive Majorana neutrino ν_j with the mass m_j in the range

$$\sqrt{s_1^-} \approx 245\mathrm{MeV} \leq m_j \leq \sqrt{s_1^+} \approx 389\mathrm{MeV}. \tag{9}$$

Then the s-channel diagram in Fig.1(a) blows up because the integrand of the first term in Eq. (1) has a non-integrable singularity at $s = m_j^2$. Therefore, in this resonant domain the total ν_j-neutrino decay width $\Gamma_{\nu j}$ has to be taken into account. This can be done by the substitution $m_j \to m_j - (i/2)\Gamma_{\nu j}$. The total decay width $\Gamma_{\nu j}$ of the Majorana neutrino ν_j with the mass in the resonant domain (9) receives contributions from the following decay modes:

$$\nu_j \longrightarrow \begin{cases} e^+\pi^-, \; e^-\pi^+, \; \mu^+\pi^-, \; \mu^-\pi^+ \\ e^+e^-\nu_e^c, \; e^+\mu^-\nu_\mu^c, \; \mu^+e^-\nu_e^c, \; \mu^+\mu^-\nu_\mu^c \\ e^-e^+\nu_e, \; e^-\mu^+\nu_\mu, \; \mu^-e^+\nu_e, \; \mu^-\mu^+\nu_\mu \end{cases}, \tag{10}$$

Since $\nu_j \equiv \nu_j^c$ it can decay in both $\nu_j \to l^- X(\Delta L = 0)$ and $\nu_j \to l^+ X^c(\Delta L = 2)$ channels. Calculating partial decay rates we obtain

$$\Gamma(\nu_j \to l\pi) = |U_{lj}|^2 \frac{G_F^2}{4\pi} f_\pi^2 m_j^3 F(y_l, y_\pi) \equiv |U_{lj}|^2 \Gamma_2^{(l)}, \tag{11}$$

$$\Gamma(\nu_j \to l_1 l_2 \nu) = |U_{l_1 j}|^2 \frac{G_F^2}{192\pi^3} m_j^5 H(y_{l1}, y_{l2}) \equiv |U_{l_1 j}|^2 \Gamma_3^{l_1 l_2}, \tag{12}$$

where $y = m_i/m_j$ and

$$F(x,y) = \lambda^{1/2}(1, x^2, y^2)[(1+x^2)(1+x^2-y^2) - 4x^2], \tag{13}$$

$$H(x,y) = 12 \int\limits_{z_1}^{z_2} \frac{dz}{z}(z-y^2)(1+x^2-z)\lambda^{1/2}(1,z,x^2)\lambda^{1/2}(0,y^2,z). \tag{14}$$

The integration limits are $z_1 = x_{l_2}^2$, $z_2 = (1 - x_{l_1})^2$. Note that $F(0,0) = H(0,0) = 1$. Summing up all the decay modes in (10) one gets for the total ν_j width

$$\begin{aligned} \Gamma_{\nu j} &= 2|U_{\mu j}|^2(\Gamma_2^{(\mu)} + \Gamma_3^{(\mu e)} + \Gamma_3^{(\mu\mu)}) + 2|U_{ej}|^2(\Gamma_2^{(e)} + \Gamma_3^{(ee)} + \Gamma_3^{(e\mu)}) \equiv \\ &\equiv |U_{\mu j}|^2 \Gamma_\nu^{(\mu)} + |U_{ej}|^2 \Gamma_\nu^{(e)}. \end{aligned} \tag{15}$$

In the resonant domain (9) $\Gamma_{\nu j}$ reaches its maximum value at $m_j = \sqrt{s_1^+}$. Assuming for the moment $|U_{\mu j}| = |U_{ej}| = 1$, we estimate this maximum value to be $\Gamma_{\nu j} \approx 1.2 \cdot 10^{-11}$MeV. Since $\Gamma_{\nu j}$ is so small in the resonant domain (9) the neutrino propagator in the first term of Eq. (1) has a very sharp maximum at $s = m_j^2$. The second term, being finite in the limit $\Gamma_{\nu j} = 0$, can be neglected in the considered case. Thus, with a good precision we obtain from Eq.(1)

$$\Gamma^{res}(\mathrm{K}^+ \to \mu^+\mu^+\pi^-) \approx \frac{c\pi}{m_K^2} G(z_0) \frac{m_j |U_{\mu j}|^4}{|U_{\mu j}|^2 \Gamma_\nu^{(\mu)} + |U_{ej}|^2 \Gamma_\nu^{(e)}} \tag{16}$$

with $z_0 = (m_j/m_K)^2$. This Eq. allows one to derive, from the experimental bound Eq. (8), the constraints on ν_j neutrino mass m_j and the mixing matrix elements $U_{\mu j}, U_{ej}$ in a form of a 3-dimensional exclusion plot. However one may reasonably assume that $|U_{\mu j}| \sim |U_{ej}|$. Then from the experimental bound (8) we derive a 2-dimensional $m_j - |U_{\mu j}|^2$ exclusion plot given in Fig. 2. For comparison we also present in Fig. 2 existing bounds taken from [15]. As seen the experimental data on the $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay exclude previously unrestricted domain. The constraints can be summarized as

$$|U_{\mu j}|^2 \leq (5.6 \pm 1) \cdot 10^{-9} \qquad \text{at} \quad 245\text{Mev} \leq m_j \leq 385\text{MeV}, \tag{17}$$

The best limit $|U_{\mu j}|^2 \leq 4.6 \cdot 10^{-9}$ is achieved at $m_j \approx 300$MeV. Note that these limits are compatible with our assumption that $|U_{\mu j}| \sim |U_{ej}|$ since in this mass domain typically $|U_{ej}|^2 \leq 10^{-9}$ [16].

The constraints from $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ in Fig. 2 and Eq. (17) can be significantly improved in the near future by experiments E949 at BNL and E950 at FNAL [17]. Important note is that in the resonant domain we have $\Gamma^{res}(\mathrm{K}^+ \to \mu^+\mu^+\pi^-) \sim |U|^2$ while outside $\Gamma(\mathrm{K}^+ \to \mu^+\mu^+\pi^-) \sim |U|^4$. Thus in the resonant mass domain the $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay has a significantly better sensitivity to the neutrino mixing matrix element. In forthcoming experiments the upper bound on the ratio in Eq. (8) can be improved by two orders of magnitude or even more. Then this experimental bound could be translated to the limit $|U_{\mu j}|^2 < 10^{-11}$ and stronger.

4 Conclusion

We studied the potential of the K-meson neutrinoless double muon decay as a probe of the Majorana neutrino masses and mixings. We found that this process is very sensitive to the MeV neutrinos ν_j in the resonant mass range (9). We analyzed the contribution of these neutrinos to the $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay rate and found from the current experimental data stringent upper limits on the Majorana neutrino mixing matrix element $|U_{\mu j}|^2$. In Fig.2 we presented these limits in the form of a 2-dimensional exclusion plot, and compared with existing limits. The $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay excludes a domain previously unrestricted experimentally. We point out that the current and near future experimental searches for this decay are not able to provide any information on the light eV-mass or heavy GeV-mass neutrinos, since in this case the required experimental sensitivities are by many orders of magnitude far from the realistic ones. Note finally that the $K^+ \to \mu^+\mu^+\pi^-$ decay can, in principle, probe the lepton number violating interactions beyond the standard model. A well known example is given by the R-parity violating interactions in supersymmetric models. However these aspects of the $K^+ \to \mu^+\mu^+\pi^-$ decay are yet to be studied.

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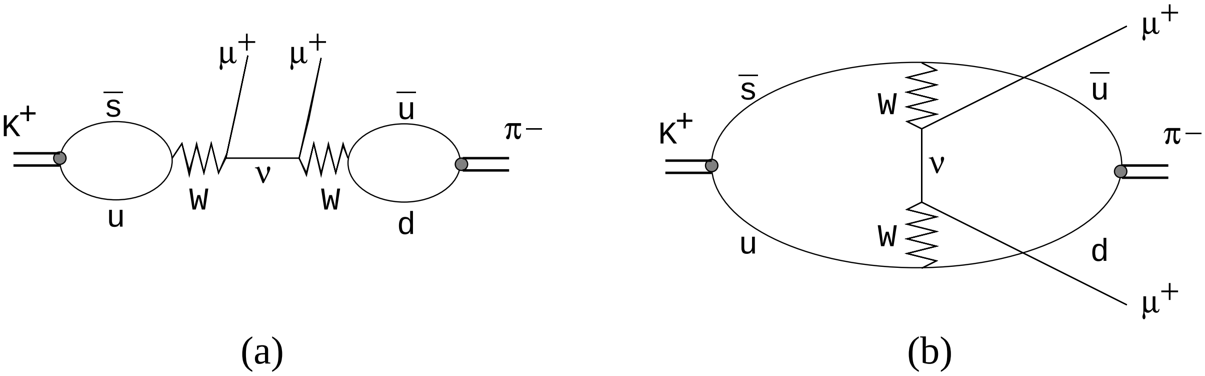

Figure 1: The lowest order diagrams contributing to $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay.

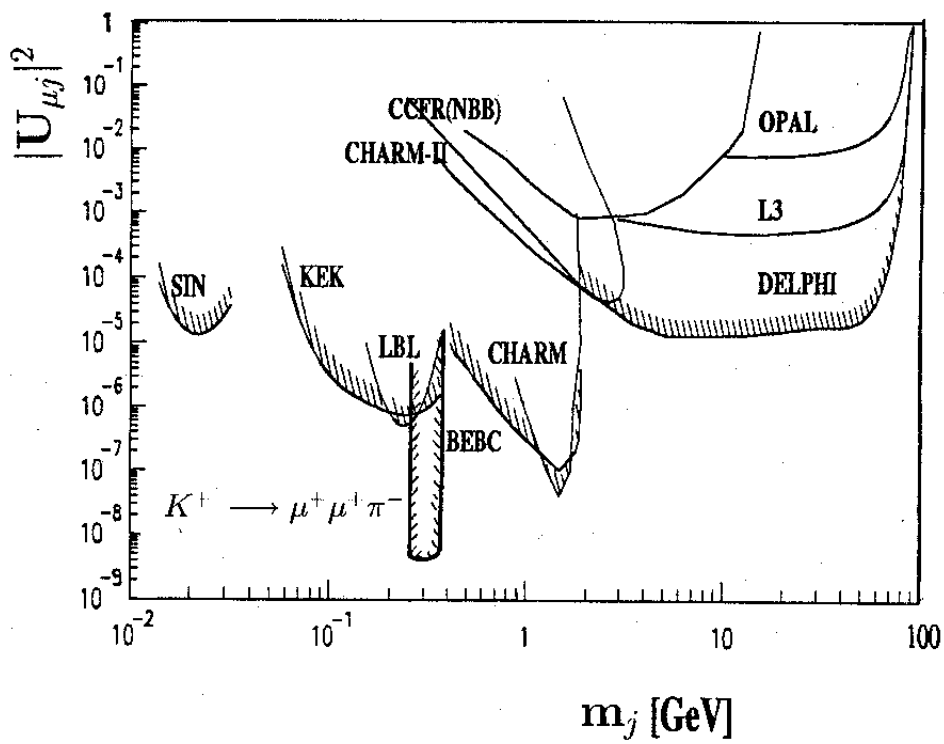

Figure 2: Exclusion plots in the plane $|U_{\mu j}|^2 - m_j$. Here $U_{\mu j}$ and m_j are the heavy neutrino ν_j mixing matrix element to ν_μ and its mass respectively. Domains above the curves are excluded by various experiments (see for recent update [15]). Region excluded by $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay covers interval 249MeV$\leq m_j \leq$ 385MeV and extends down to $|U_{\mu j}|^2 \leq 4.6 \cdot 10^{-9}$.